SM 3H

Review for Test 1.1-1.8 KEY

2017-2018

Name _____ Date _____ Period _____

Describe the end behavior of each polynomial using limit notation, without using a graphing calculator.

1. $f(x)=(x+3)(x-1)(2x-5)$ degree 3

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

2. $f(x)=-(x+4)^2(x-2)$ Degree 3

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

3. $f(x)=-2x^3-3x^2+36x-58$ degree 3

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

4. $f(x)=3x^4-7x^3+16x^2-15x+65$

degree 4

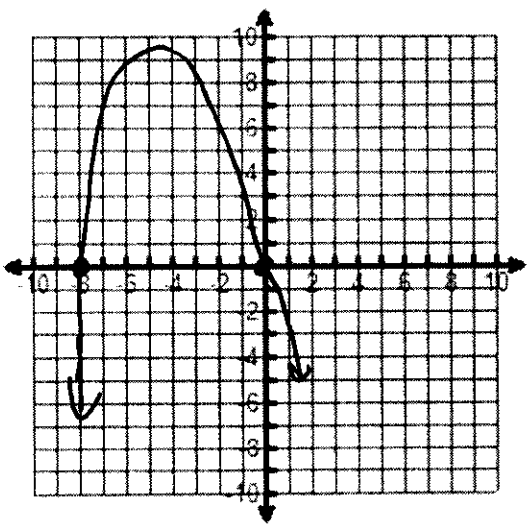
$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

State the degree and list the zeros of the polynomial. State the multiplicity of each zero and determine whether the graph crosses or touches the x-axis at the corresponding x-intercept. Then sketch a graph.

5. $f(x)=-2x^3(x+8)$ Degree: 4

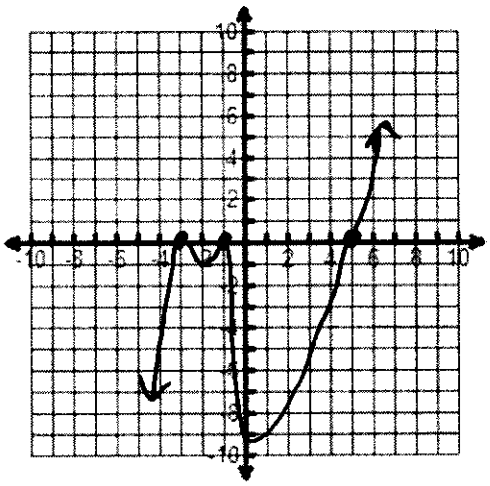
Zero	Multiplicity	Touch/Cross
x=0	3	cross
x=-8	1	Cross



$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = -\infty$

6. $f(x)=(x+1)^2(x-5)^3(x+3)^2$ Degree: 7

Zero	Multiplicity	Touch/Cross
$x=-1$	2	Touch
$x=5$	3	cross
$x=-3$	2	Touch



$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = \infty$

Multiply the expression using the polynomial identities.

7. $(3x+2y)^2$

$(3x)^2+2(3x)(2y)+(2y)^2$

$9x^2+12xy+4y^2$

$A^3+3A^2B+3AB^2+B^3$

8. $(x-2y)^3$

$A=x$

$B=-2y$

$(x)^3+3(x)^2(-2y)+3(x)(-2y)^2+(-2y)^3$

$x^3-6x^2y+12xy^2-8y^3$

9. $(x-4)(x+6)$

$x^2+(-4+6)x+(-4)(6)$

$x^2+2x-24$

10. $(5x+i)(5x-i)$ $(A+Bi)(A-Bi)=A^2+B^2$

$A=5x$

$B=1$

$(5x)^2+(1)^2$

$25x^2+1$

Factor the expressions using the polynomial identities.

11. $25x^2-64$

$(5x-8)(5x+8)$

$A=x, B=5$

12. x^3+125

$(x+5)(x^2-5x+25)$

13. $x^2-4x-21$

$(x-7)(x+3)$

14. $9x^2+81$

$(3x+9i)(3x-9i)$

Use the quadratic formula to solve each equation. Show work!

15. $x^2 - 17x = -72$

$x^2 - 17x + 72 = 0$

$x = \frac{17 \pm \sqrt{(-17)^2 - 4(1)(72)}}{2}$

$x = \frac{17 \pm \sqrt{289 - 288}}{2} = \frac{17 \pm 1}{2}$

$x = \frac{17+1}{2} = \frac{18}{2} = 9$

$x = \frac{17-1}{2} = \frac{16}{2} = 8$

$x = 9$, $x = 8$

16. $5x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(1)}}{2(5)}$

$x = \frac{3 \pm \sqrt{9 - 20}}{10}$

$x = \frac{3 \pm \sqrt{11}\, i}{10}$

Simplify the expression. Show work!

17. $(x+5)(2x-1) - (3x^2 - 16x + 3)$

$2x^2 - x + 10x - 5 - 3x^2 + 16x - 3$

$-x^2 + 25x - 8$

Divide *f(x)* by *d(x)* using long division. *W*rite answer in fraction form. According to the Factor Theorem, is $d(x)$ a factor of $f(x)$? Show work!

18. $f(x) = 2x^3 - 3x^2 + 4x - 8$, $d(x) = x - 1$ Yes or (No)

$$x-1 \overline{)\, 2x^3 - 3x^2 + 4x - 8}$$

$2x^2 - x + 3 + \frac{-5}{x-1}$

$-(2x^3 - 2x^2)$

$-x^2 + 4x$

$-(-x^2 + x)$

$3x - 8$

$-(3x - 3)$

-5

Remainder is not zero.

$2x^2 - x + 3 + \frac{-5}{x-1}$

Divide using synthetic division. Write answer in fraction form. Show work!

19. $\frac{2x^3 + 3x^2 + 4x - 10}{x+1}$

-1	2	3	4	-10
	↓	-2	-1	-3
	2	1	3	-13

$2x^2 + x + 3 + \frac{-13}{x+1}$

Write an equation in factored form and standard form for the function with the given zeros. Show work!

20. $x=3,\ x=-5,\ x=0$

Factored Form: $x(x-3)(x+5)=0$

$x(x-3)(x+5) = x(x^2+2x-15)$
$= x^3+2x^2-15x$

Standard Form: $x^3+2x^2-15x=0$

Factor to find the zeros of each of the following polynomials.

21. $f(x)=x^3+4x^2+3x$

$x(x^2+4x+3)$
$x(x+3)(x+1)=0$
$x=0,\ x=-3,\ x=-1$

22. $f(x)=x^2-6x-16$

$(x-8)(x+2)=0$
$x=8,\ x=-2$

Use the Rational Zeros Theorem to write a list of all potential rational zeros. Show work!

22. $f(x)=3x^3+43x^2+43x+27$

$\frac{\text{factors of } 27}{\text{factors of } 3} = \frac{\pm1, \pm3, \pm9, \pm27}{\pm1, \pm3}$

$\pm1, \pm\frac{1}{3}, \pm3, \pm9, \pm27$

Using the given zero, find all of the zeros and write a factored form of *f(x)*. Show work!

23. $3i$, is a zero of $f(x)=x^4-x^3+7x^2-9x-18$

$x=3i,\ x=-3i$
$(x-3i)(x+3i) = x^2+9$

$f(x)=(x+3i)(x-3i)(x-2)(x+1)$

$x^2+9 \overline{)\ x^4-x^3+7x^2-9x-18}$ $\quad x^2-x-2$

$-(x^4 \quad +9x^2)$
$-x^3-2x^2-9x$
$-(-x^3 \quad -9x)$
$-2x^2-18$
$-(-2x^2-18)$
0

x^2-x-2 (factor)
$(x-2)(x+1)$

Zeros: $x=3i,\ x=-3i$
$x=2,\ x=-1$

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed. Show work!

24. $x=-2,\ x=1+2i$, $x = 1-2i$

$(x+2)(x-1-2i)(x-1+2i)$ $\quad 1^2+2^2=5$

$(x+2)(x^2-2x+5)$

$x^3-2x^2+5x+2x^2-4x+10$

$\boxed{f(x)=x^3+x+10}$

25. Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros of the polynomial function. Hint: Use *f*(*x*) to find number of positives and *f*(-*x*) to find number of negatives.

$f(x)=-x^2-5x-8$ no sign changes so none

Number of possible positive zeros: no possible positive real zeros.

$f(-x)=$ $-(-x)^2-5(-x)-8=-x^2+5x-8$ (2 sign changes)

Number of possible negative zeros: 2 or 0

26. Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational. Show work!

$f(x)=x^3+x^2-2x-2$

Factor by grouping:

x^3+x^2-2x-2

$x^2(x+1)-2(x+1)$

$(x^2-2)(x+1)$

$x^2-2=0$ $\quad x+1=0$

$x^2=2$ $\quad x=-1$

$x=\pm\sqrt{2}$

Zeros	Rational/Irrational
$x=-1$	rational
$x=\sqrt{2}$	irrational
$x=-\sqrt{2}$	irrational

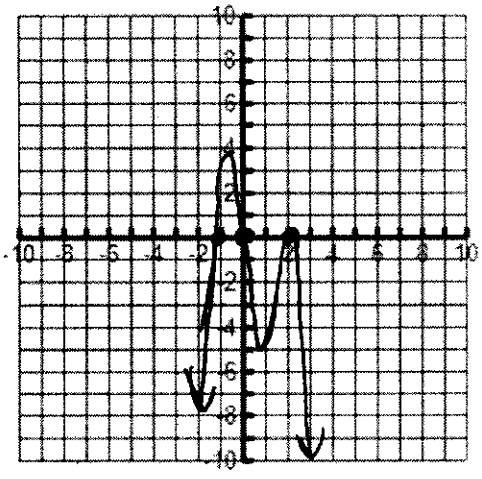
27. Write a polynomial function of minimum degree in <u>factored form</u> with real coefficients whose zeros and their multiplicities include those listed. Find the <u>degree of the polynomial</u>, the <u>x-intercepts</u>, <u>end behaviors</u> (using limit notation) and <u>sketch the graph.</u> Show work!

leading coefficient: -3

$x = 0$ (multiplicity 1) x,

$x = 2$ (multiplicity of 2), $(x-2)^2$

$x = -1$ (multiplicity of 1) $(x+1)$



Factored form: $f(x) = -3x(x-2)^2(x+1)$

Degree: 4

End behaviors (write using limit notation):

$\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = -\infty$

x-intercepts (write as ordered pair): $(0,0)(2,0)(-1,0)$

↓ cross ↓ touch ↓ cross